Following the design procedure, we let $j = 1$. The input weighting matrix is specified as $R = \hat{R} = \mathrm{diag}\,[3,1]$. According to our notation, $(\lambda_i^-) = 0.4$ and $(\lambda_{1,2}^+) = 0.5 \pm j0.7$. From the inverse of the block modal matrix M, we obtain

$$\hat{M}_y = \begin{bmatrix} 0.1712 & -0.0112 & -0.28 \\ 0.1584 & -0.0384 & 0.04 \end{bmatrix} \tag{15}$$

To relocate the poles $\lambda_{2,3}$ on the positive real axis, we choose the column vector $\boldsymbol{d}$ as $\boldsymbol{d} = [1,0]^T$, so that the asymptotic poles of the closed-loop system tend to 0 and 0.6225 as $q \to \infty$, i.e.,

$$\beta(z)\beta^T(z^{-1}) = 0.103(z - 0.4)(z^{-1} - 0.4) \times (z - 0.6225)(z^{-1} - 0.6225)$$

Selecting $q = 60$, the variant closed-loop poles become $\alpha_2 = 0.215$ and $\alpha_3 = 0.45$. The state weighting matrix Q is

$$Q = q\hat{M}_y^T \boldsymbol{dd}^T \hat{M}_y = \begin{bmatrix} 1.76 & -0.115 & -2.88 \\ -0.115 & 0.0075 & 0.19 \\ -2.88 & 0.19 & 4.7 \end{bmatrix} \tag{16}$$

For the closed-loop eigenvalue $\alpha_2 = 0.215$, the vectors ρ_2 and ξ_2 are $\rho_2 = [-0.312,1]^T$ and $\xi_2 = [-3.627, -5.312, -1.075]^T$. Similarly, for $\alpha_3 = 0.45$, we have $\rho_3 = [-0.3,1]^T$ and $\xi_3 = [-2.0,4.0,-0.925]^T$. Thus, from Eq. (8b), the state feedback gain K becomes

$$K = -[0_{2\times 1},\rho_2,\rho_3][M_1^-,\xi_2,\xi_3]^{-1} = \begin{bmatrix} -0.15 & 0.023 & 0.1 \\ 0.466 & -0.077 & -0.26 \end{bmatrix} \tag{17}$$

where $M_1^- = [1.4,6.4,0.6]^T$. Note that this feedback gain can also be obtained by solving the discrete Riccati equation in Eq. (4), with (A,B) and (Q,R). The optimal closed-loop system is

$$A := A - BK = \begin{bmatrix} -0.2 & 0 & 1.4 \\ -0.75 & 0.423 & 1.5 \\ -0.184 & 0 & 0.84 \end{bmatrix} \tag{18}$$

with its eigenvalues being $\sigma(A_c) = \{0.215,0.4,0.45\}$. The final closed-loop system has positive real eigenvalues at desired locations and it is optimal with respect to the performance index in Eq. (2) for (Q,R).

V. Conclusion

A sequential method that uses the classical root-locus techniques has been developed to determine the quadratic weighting matrices and the discrete linear quadratic regulators of multivariable control systems. In this proposed approach, at each recursive step, an intermediate unity rank state-weighting matrix containing some invariant eigenvectors of that open-loop system matrix is assigned. Also, at each step, an intermediate characteristic equation of the closed-loop system containing the invariant eigenvalues is created. In order to control the movement of the root-loci and choose desirable closed-loop poles, some virtual finite open-loop zeros are assigned to this characteristic equation. The designed optimal closed-loop system thus would retain some stable open-loop poles and have the remaining poles optimally placed at desired locations in the complex z plane.

Acknowledgments

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Problem of the Dynamics of a Cantilever Beam Attached to a Moving Base

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Introduction

In a recent article in this journal, Kane et al.[1] formulated a comprehensive theory of a cantilever beam mounted on a moving support. They took into account the stretch, bending in two principal directions, shear deformations, and warping of the beam. The formulation is based on a method that has been attributed to Kane. Kane's method,[2] which is closely related to Gibbs' method, provides a systematic procedure and

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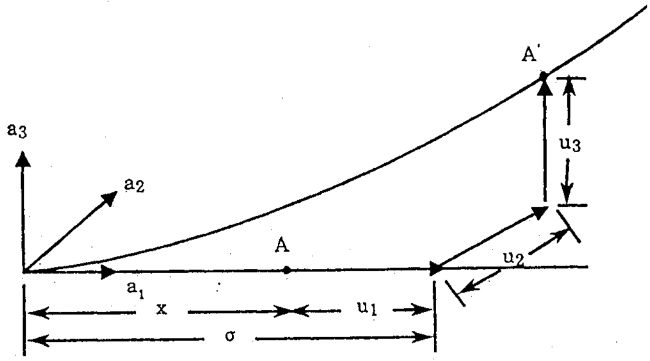

Fig. 1a Displacements at neutral axis.

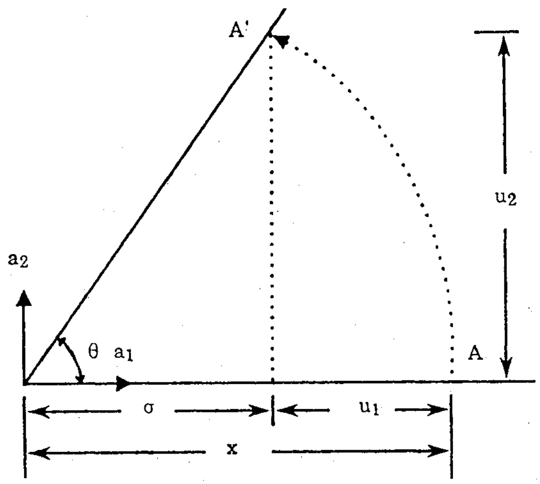

Fig. 1b Displacements of a rigid bar.

has the potential of yielding a concise and simple form of resulting equations of motion. In Ref. 1, the important effect of centrifugal stiffening has been included without making any ad hoc assumptions. The transient dynamics of a spinning bar starting from rest were studied to illustrate some important results. They showed how some of the existing formulations lead to unrealistic solutions when centrifugal effects are not included properly. They took the stretch of the beam to be an independent kinematical quantity and associated the centrifugal stiffening effect with nonzero axial motion for every nonzero transverse motion. We, however, believe that axial and transverse motion can be totally independent, and that stable dynamic response of a rotating beam is observed when the nonlinear effects are properly included in the formulation. We are also of the opinion that the stiffening terms in Ref. 1 were obtained with an inconsistent stretch expression. In this Note, we point out this inconsistent use of the stretch term and present an alternative and consistent formulation of a beam attached to a moving base by using Kane's method of dynamics.

This work differs from Ref. 1 in the following ways. Instead of stretch as an independent variable, axial displacement along with bending displacements in two transverse directions have been selected as independent variables. The present study considers large displacements and rotations by incorporating nonlinear strain displacement relations into the formulation. Terms up to cubic order have been retained in the final dynamical equations. The axial and transverse displacements have been written in terms of mode shapes and generalized coordinates. The generalized coordinates have been chosen independently for each displacement, unlike in Ref. 1, where a common set of generalized coordinates was used. For numerical studies, the new formulation, based on Kane's method, is used to study the same spin-up problem of a beam starting from rest. The effect of nonlinear structural terms on the stable behavior of the beam is discussed.

Stretch of the Beam

The deformation of the neutral axis is sketched in Fig. 1a. The neutral axis lies parallel to the a_1 axis prior to deformation. Point A on the neutral axis is identified by its distance x from the origin in the undeformed state. The axial displacement u_1 and the transverse displacements u_2 and u_3 can be written as functions x. The stretch at any location can be obtained as follows. The relationship between deformed length ds and undeformed length dx is given by

$$\frac{ds^2 - dx^2}{2\ dx^2} = u_{1,x} + \tfrac{1}{2}[u_{1,x}^2 + u_{2,x}^2 + u_{3,x}^2] \tag{1}$$

Using Eq. (1), the stretch $s(x_1)$ at any point x_1 can be written as

$$x_1 + s(x_1,t) = \int_0^{x_1} [(1 + u_{1,x})^2 + u_{2,x}^2 + u_{3,x}^2]^{1/2}\ dx \tag{2}$$

This expression for stretch is different from that given in Ref. 1 in two ways. First, in the integrand, the derivatives of the axial displacement appear explicitly; second, the limits are different. The stretch in Ref. 1 can be obtained from Eq. (2) by a simple transformation. In Fig. 1a, let σ be the projection of the deformed neutral axis on the a_1 axis. The transverse displacements can be written as functions of σ and are denoted by caret quantities to distinguish them from the displacement functions written as functions of x. We have

$$\hat{u}_2(\sigma) = u_2(x); \quad \hat{u}_3(\sigma) = u_3(x); \quad \sigma(x) = x + u_1(x) \tag{3}$$

Using these relations, stretch given by Eq. (2) can be transformed to obtain the stretch expression given in Ref. 1:

$$x_1 + s(x_1,t) = \int_0^{\sigma_1} [1 + \hat{u}_{2,\sigma}^2 + \hat{u}_{3,\sigma}^2]^{1/2}\ d\sigma \tag{4}$$

If the displacements are expressed in terms of the undeformed coordinate x, stretch should be obtained by using Eq. (2). If the displacements are expressed in terms of the deformed coordinate σ, then Eq. (4) should be used for stretch. Both of the stretch expressions given by Eqs. (2) and (4) can be used as long as the displacements are expressed in terms of the proper independent variables. To illustrate, we consider a rigid bar, whose motion takes place in one plane (Fig. 1b). If the displacements are expressed in terms of x, we have

$$u_1(x) = x(\cos\theta - 1); \quad u_2(x) = x \sin\theta; \quad u_3(x) = 0 \tag{5}$$

Substituting these displacements in the stretch expression given by Eq. (2), we get zero stretch for a rigid bar. On the other hand, if displacements are expressed in terms of the projected coordinate σ, we have

$$\hat{u}_2(\sigma) = \sigma \tan\theta; \quad \hat{u}_3(\sigma) = 0 \tag{6}$$

Substituting these in Eq. (4), we again get zero stretch for a rigid bar. It is to be noted that stretch will not be zero if the transverse displacements given by Eq. (5) are used in the stretch expression given by Eq. (4). In Ref. 3, Kane has clarified that, in Eq. (19) of Ref. 1, σ is the x coordinate of a generic point of the neutral axis when the beam is in the deformed state. In that case, the transverse displacements should have been written in terms of the deformed coordinate σ; in Eq. (26) of Ref. 1 the transverse displacements have been expressed as functions of the undeformed coordinate x. Then Eq. (4) cannot be used to obtain the stretch. This stretch term has been used in Ref. 1 to construct partial velocities, which ultimately yield the stiffness terms in the dynamical equations. Therefore, the stable response of the beam in Ref. 1 is due to an inconsistent use of the stretch expression. A consistent formulation has been presented in this Note.

Formulation

In this section, dynamical equations have been derived for the independent transverse and axial displacements. The mass and moments of inertia of the base are assumed to be zero. The cross sections are assumed to remain normal after deformation. The effects of warping and rotary inertia are not included in this formulation. Since the displacements could be large, nonlinear strain-displacement relationships are used to determine the strain energy function of the beam and terms up to fourth order are retained. The generalized active forces due to elastic deformation have been determined by differentiation of the strain energy function, and the generalized inertial forces have been found by taking the dot products between accelerations and partial velocities.

Figure 1a is a sketch of the beam after deformation. The quantities $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, and $\boldsymbol{a}_3$ are orthogonal unit vectors that are attached to the base, and they undergo the same inertial motion as that of the base. The quantities v_1, v_2, v_3 and w_1, w_2, w_3 are the translation and rotational velocity components, respectively, of the base along $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, and $\boldsymbol{a}_3$. When the beam is in its undeformed state, the neutral axis lies along $\boldsymbol{a}_1$ and points are identified by their distance from the origin and denoted by x. The beam displacements are given in the rotating frame and are expressed as functions of x. Then, the assumed displacements take the following form:

$$u_j(x,t) = \sum_{i=1}^{N_j} \phi_{ji}(x,t)\, q_{ji}(t), \qquad j = 1,2,3 \tag{7}$$

It is to be noted that different sets of generalized coordinates are chosen for every displacement. Velocity and acceleration are derived from Eq. (7) and, subsequently, generalized inertial forces are obtained. The generalized active forces caused by elastic effects are determined by differentiating the strain energy function. The strain energy function U is written as

$$U = \frac{1}{2}\int_0^l [EI_2 u_{3,xx}^2 + EI_3 u_{2,xx}^2 + EAu_{1,x}^2 + EAu_{1,x}(u_{2,x}^2 + u_{3,x}^2) + \frac{1}{4}EA(u_{2,x}^4 + u_{3,x}^4 + 2u_{2,x}^2 u_{3,x}^2)]\,\mathrm{d}x \tag{8}$$

where E is Young's modulus, A the area of the cross section, and I_2 and I_3 are the area moments of inertias of the cross section about the $\boldsymbol{a}_2$ and $\boldsymbol{a}_3$ axes, respectively. The dynamical equations are obtained by using Kane's method. The axial equations corresponding to the generalized speed $\dot{q}_{1i}$ are written as

$$\begin{aligned}
&\sum_{i=1}^{N_1} W_{11ki}\ddot{q}_{1i} + \sum_{i=1}^{N_1} K_{11ki}q_{1i} + \frac{1}{2}\sum_{m=1}^{N_2}\sum_{n=1}^{N_2} G_{122kmn}q_{2m}q_{2n} \\
&+ \frac{1}{2}\sum_{m=1}^{N_3}\sum_{n=1}^{N_3} G_{133kmn}q_{3m}q_{3n} + \left(\dot{w}_2 + w_2w_3\right)\sum_{i=1}^{N_3} W_{13ki}q_{3i} \\
&+ \left(-\dot{w}_3 + w_1w_2\right)\sum_{i=1}^{N_2} W_{12ki}q_{2i} - \left(w_2^2 + w_3^2\right)\sum_{i=1}^{N_1} W_{11ki}q_{1i} \\
&+ 2w_2\sum_{i=1}^{N_3} W_{13ki}\dot{q}_{3i} - 2w_3\sum_{i=1}^{N_2} W_{12ki}\dot{q}_{2i} \\
&= -(\dot{v}_1 + w_2v_3 - w_3v_2)W_{1k} + (w_2^2 + w_3^2)X_{1k}
\end{aligned} \tag{9}$$

The transverse dynamical equation corresponding to $\dot{q}_{2i}$ is written as

$$\begin{aligned}
&\sum_{i=1}^{N_2} W_{22ki}\ddot{q}_{2i} + \sum_{i=1}^{N_2} K_{22ki}q_{2i} + \sum_{i=1}^{N_1}\sum_{m=1}^{N_2} G_{122ikm}q_{1i}q_{2m} \\
&+ \left(\dot{w}_3 + w_1w_2\right)\sum_{i=1}^{N_1} W_{21ki}q_{1i} - \left(w_1^2 + w_3^2\right)\sum_{i=1}^{N_2} W_{22ki}q_{2i} \\
&+ \left(-\dot{w}_1 + w_2w_3\right)\sum_{i=1}^{N_3} W_{23ki}q_{3i} + 2w_3\sum_{i=1}^{N_1} W_{21ki}\dot{q}_{1i} \\
&- 2w_1\sum_{i=1}^{N_3} W_{23ki}\dot{q}_{3i} + \frac{1}{2}\sum_{i=1}^{N_2}\sum_{j=1}^{N_2}\sum_{l=1}^{N_2} E_{2222kijl}q_{2i}q_{2j}q_{2l} \\
&+ \frac{1}{2}\sum_{i=1}^{N_2}\sum_{j=1}^{N_3}\sum_{l=1}^{N_3} E_{2233kijl}q_{2i}q_{3j}q_{3l} \\
&= -(\dot{v}_2 + w_3v_1 - w_1v_3)W_{2k} - (\dot{w}_3 + w_1w_2)X_{2k}
\end{aligned} \tag{10}$$

Similarly, the equation corresponding to $\dot{q}_{3i}$ can be obtained. In these equations the following quantities have been defined:

$$W_{mnij} = \int_0^l \rho\phi_{mi}(x)\phi_{nj}(x)\,\mathrm{d}x$$

$$m,n = 1,2,3; \quad i = 1,\ldots,N_m; \quad j = 1,\ldots,N_n \tag{11a}$$

$$K_{22ij} = \int_0^l EI_3\phi_{2i}''(x)\phi_{2j}''(x)\,\mathrm{d}x$$

$$i = 1,\ldots,N_2; \quad j = 1,\ldots,N_2 \tag{11b}$$

$$K_{33ij} = \int_0^l EI_2\phi_{3i}''(x)\phi_{3j}''(x)\,\mathrm{d}x$$

$$i = 1,\ldots,N_3; \quad j = 1,\ldots,N_3 \tag{11c}$$

$$K_{11ij} = \int_0^l EA\,\phi_{1i}'(x)\phi_{1j}'(x)\,\mathrm{d}x$$

$$i = 1,\ldots,N_1; \quad j = 1,\ldots,N_1 \tag{11d}$$

$$W_{pi} = \int_0^l \rho\phi_{pi}(x)\,\mathrm{d}x; \qquad p = 1,2,3; \quad i = 1,\ldots,N_p \tag{11e}$$

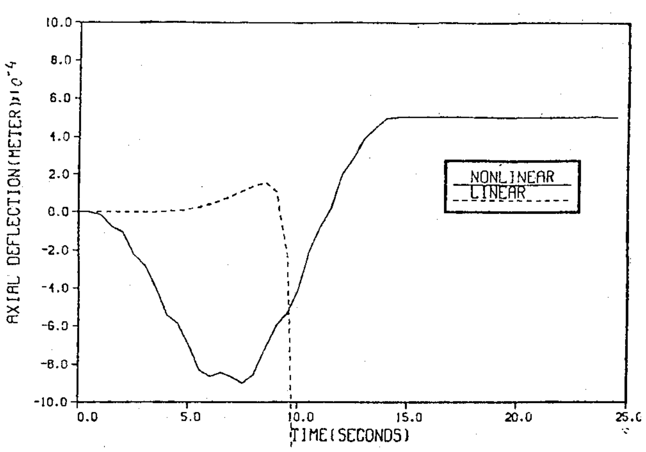

Fig. 2a Axial displacement vs time.

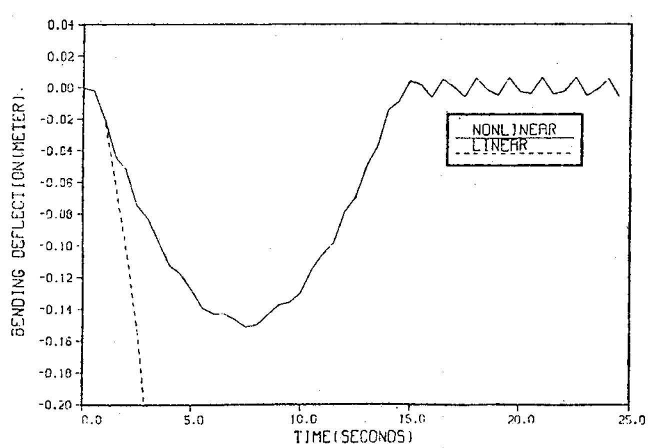

Fig. 2b Bending displacement vs time.

$$X_{pi} = \int_0^l \rho x \phi_{pi}(x)\, dx; \qquad p = 1,2,3; \quad i = 1,\ldots,N_p \tag{11f}$$

$$G_{122imn} = \int_0^l EA\, \phi_{1i}'(x)\phi_{2m}'(x)\phi_{2n}'(x)\, dx$$

$$i = 1,\ldots,N_1; \quad m,n = 1,\ldots,N_2 \tag{11g}$$

$$G_{133imn} = \int_0^l EA\, \phi_{1i}'(x)\phi_{3m}'(x)\phi_{3n}'(x)\, dx$$

$$i = 1,\ldots,N_1; \quad m,n = 1,\ldots,N_3 \tag{11h}$$

$$E_{ppqqijkl} = \int_0^l EA\, \phi_{pi}'(x)\phi_{pj}'(x)\phi_{qk}'(x)\phi_{ql}'(x)\, dx$$

$$p,q = 2,3; \quad i,j = 1,\ldots,N_p; \quad k,l = 1,\ldots,N_q \tag{11i}$$

Results and Discussion

The spin-up problem in Ref. 1 is also considered here to study the effect of nonlinear structural terms. The beam is initially at rest and an angular velocity is given along the a_3 axis at the base. Only the in-plane motions are excited in line with the assumptions. The beam parameters are as follows: $E = 68{,}950{,}000{,}000\ \text{N/m}^2$, $\rho = 1.2\ \text{kg/m}^3$, $A = 0.0004601\ \text{m}^2$, $I_3 = 0.0000002031\ \text{m}^4$, and $l = 10$ m. The angular velocity history is taken to be identical with that in Ref. 1

$$w_3 = 6/15 \left[t - \frac{15}{2\pi} \sin \frac{2\pi t}{15} \right] \text{rad/s}, \qquad 0 \le t \le 15 \text{ s}$$

$$w_3 = 6 \text{ rad/s}, \qquad t \ge 15 \text{ s} \tag{12}$$

The transverse mode shapes are taken as the fixed-free nonrotating eigenfunctions of a uniform beam under transverse vibration, whereas the longitudinal modes are taken as the eigenfunctions of a fixed-free uniform rod under longitudinal vibrations. The axial and transverse motions are represented by one and three modes, respectively. The axial and bending responses of the tip of the cantilever beam resulting from the formulation presented in this Note are shown in Figs. 2a and 2b, respectively. The solid curves correspond to the analysis of this Note, where all higher-order terms have been retained, and the dashed curves refer to the situation where all second- and third-order terms are eliminated. In the nonlinear analysis, the transverse deflection grows initially in a direction opposite to that of the base motion. After reaching a maximum displacement, the tip goes back toward the equilibrium position and settles down to a steady oscillation. The nonlinear stiffening action in the beam prevents it from going unstable, and the very absence of such terms causes the linear beam to diverge away from the equilibrium point. A foreshortening effect is observed in the axial response (Fig. 2a) when the beam is started from rest. The final axial response in the case of nonlinear analysis is a steady oscillation about a nonzero equilibrium point that corresponds to the steady-state axial displacement under the centrifugal force field. The effect of centrifugal stiffening has been studied by numerous investigators in the past, and a brief discussion on the subject can be found in Ref. 4. Likins et al.[5] have assumed steady-state axial displacements and moderate rotations to obtain the stiffening effect. Vigneron[6] has assumed foreshortening of the beam and uses Hamilton's principle to show that the centrifugal stiffening terms arise from the kinetic energy terms. In Ref. (4), Kaza and Kvaternik have observed that foreshortening of the beam need not be considered explicitly if terms up to fourth order are retained in the energy terms. The present formulation has taken such an approach and does not assume any a priori kinematical restriction on the displacement field. The axial and transverse displacement fields have been chosen to be independent of each other, and the foreshortening of the beam is a consequence of the imposed base motion.

Conclusions

In this Note, we have formulated the problem of a cantilever beam attached to a moving support by using Kane's method. The formulation is valid for large displacements, and all geometric nonlinearities have been considered in the strain-displacement relations. The method has been validated by studying the stability characteristics of a beam under the spin-up maneuver. It has been demonstrated that structural nonlinearities play a major role in the transient response characteristics and they cannot be ignored.

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Gravitational Moment Exerted on a Small Body by an Oblate Body

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Introduction

THE gravitational forces and moments that act on an orbiting body are well recognized as important influences on the motion of such a body. This paper illustrates a method for finding an analytic expression for the moment about a body's mass center produced by gravitational forces.

Expressions for the gravitational moment exerted on a body by a sphere (or particle) appear in numerous places, including Ref. 1. The equation in Ref. 1 is particularly simple because it does not express the gravitational moment in terms of a particular unit vector basis. This simplicity is made possible by expressing the gravitational moment in terms of a unit vector and a dyadic. One can implement the equation by expressing the unit vector and dyadic in any convenient basis.

In his Engineering Note, Glandorf[2] seeks and develops a method for obtaining the gravitational moment exerted by bodies other than spheres. Kane, et al.[1] suggest an alternative to the method proposed by Glandorf and simplified by Wilcox.[3] Use of the method from Ref. 1 can lead to vector-dyadic expressions that are simple in appearance and basis-independent.

As an example of the use of the method suggested in Ref. 1, this paper derives an expression for the gravitational moment

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